

Topic 7

# Differential Equations

$$\frac{AB}{BC}$$

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB  
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

6. Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

- (a) Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .
- (b) Find the domain and range of the function  $f$  found in part (a).
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**AP Calculus AB**  
**2001 Free-Response Questions**

2001  
6. The function  $f$  is differentiable for all real numbers. The point  $\left(3, \frac{1}{4}\right)$  is on the graph of  $y = f(x)$ , and the slope at each point  $(x, y)$  on the graph is given by  $\frac{dy}{dx} = y^2(6 - 2x)$ .

(a) Find  $\frac{d^2y}{dx^2}$  and evaluate it at the point  $\left(3, \frac{1}{4}\right)$ .

(b) Find  $y = f(x)$  by solving the differential equation  $\frac{dy}{dx} = y^2(6 - 2x)$  with the initial condition  $f(3) = \frac{1}{4}$ .

**END OF EXAMINATION**

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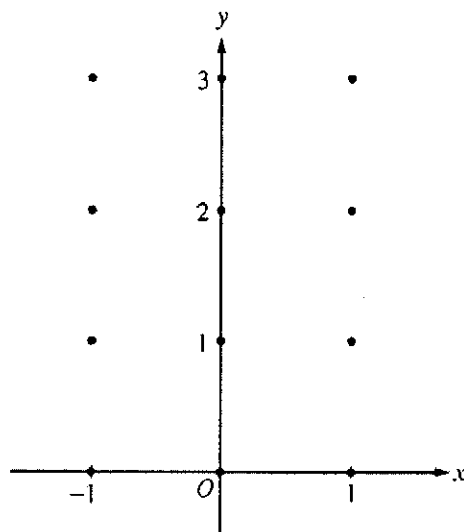
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## 2004 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation  $\frac{dy}{dx} = x^2(y - 1)$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
(Note: Use the axes provided in the pink test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane. Describe all points in the  $xy$ -plane for which the slopes are positive.
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 3$ .
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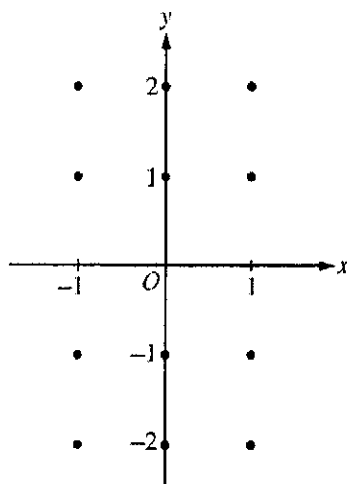
END OF EXAMINATION

## 2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation  $\frac{dy}{dx} = -\frac{2x}{y}$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(Note: Use the axes provided in the pink test booklet.)



(b) Let  $y = f(x)$  be the particular solution to the differential equation with the initial condition  $f(1) = -1$ .

Write an equation for the line tangent to the graph of  $f$  at  $(1, -1)$  and use it to approximate  $f(1.1)$ .

(c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(1) = -1$ .

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WRITE ALL WORK IN THE TEST BOOKLET.

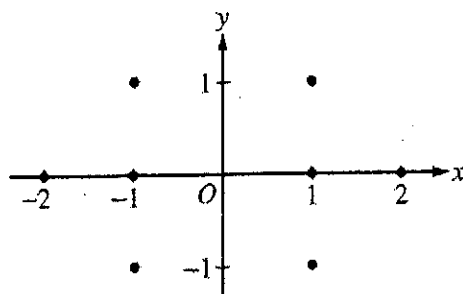
END OF EXAM

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{1+y}{x}$ , where  $x \neq 0$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)



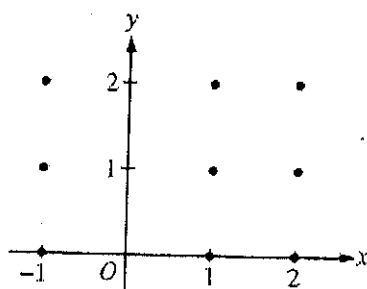
(b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(-1) = 1$  and state its domain.

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**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.  
(Note: Use the axes provided in the exam booklet.)



- (b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .  
(c) For the particular solution  $y = f(x)$  described in part (b), find  $\lim_{x \rightarrow \infty} f(x)$ .